Segregation in cities according to household size

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Summary

Over the last two centuries, household size has decreased considerably. Within a theoretical model I investigate the relationship between household size and the structure and size of cities. Household utility is assumed to depend on household size, in addition to the consumption of housing and a numeraire good. This basic building block is combined with a Muth-type urban model. The model is used for examining the impact of household size on the sorting of households according to household size, the geographical extension of the city, household utility, forms of housing etc. These issues have previously received some attention in empirical studies, but I am not aware of theoretical examinations.

1. Introduction

In industrialized countries, household size has over the last two centuries decreased considerably. This demographic transition has taken several forms. In an early phase, multi-generation households split up in separate households, and the number of children in each family declined. In a later phase, families split up in single-parent and single-person households, young people lived as singles for a longer period of their life than was usual some decades ago, and the number of elderly living as singles also increased. In the present paper, the focus is mainly on the last of these phases.

A few numbers may illustrate how dramatic the changes in household size have been over the last 50-60 years. In 1950, about 15 per cent of the Norwegian households consisted of only one person. By 2001 this share had increased to 38 per cent, and in the larger cities the share was even higher. Presumably, similar changes have occurred in other industrialized countries.

The effects that arise when multi-person households replace single-person households are quite complex, both at the household level and at the social level. At the household level, a single-person household usually will have a lower income than a larger household. If there are economies of scale in household consumption, individuals in single-person households will presumably also obtain a lower utility level than they could enjoy in a larger household. These differences are likely to result in different consumption patterns. In particular, in a city containing many single-person households a larger number of housing units will be demanded. Presumably, single-person households will also occupy smaller dwellings, located on smaller lots, than multi-person households. This will have a strong impact on the land market, population density, and the geographical extension of cities. One would also expect that households of different size would settle in separate segments of the city. This raises the question of who would live in the central parts and who at the outskirts. Another related issue of some interest is which group will live in apartments in the central parts of the city and who will live in low-density detached dwellings. To the best of my knowledge, these issues have not previously been subjected to a thorough theoretical examination, despite the fact that they have a profound impact on many aspects of people’s lives. In the present paper I therefore examine the impact on the urban housing sector of the increasing number of single-person households in urban areas.

In empirical research, the impact of household size on the urban housing sector has received some attention. For instance, Skaburskis (1999) finds that a reduction in household size leads to a drop in the demand for low-density housing. A better theoretical understanding of such relationships is however needed, inter alia in order to improve the specification of empirical models and facilitate the interpretation of the results. In addition, a better theoretical grasp of the role of household size in the

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urban housing sector is needed to guide policy decisions made by local governments. For instance, some local governments pursue an active housing policy in order to increase the number of multi-person households with children that live in the central parts of cities. It is my hope that the analysis in the present paper will prove to be useful in judging whether such policies are warranted.

In the next section, we set up a simple model of household choice between housing and a numeraire good. In fact, the only extension compared to the model suggested by Alonso (1964) is that we include household size as an explicit parameter. Within this simple model we are able to derive a condition that determines whether single-person households will reside in the central parts of the city or in the outskirts. In Section 3 we illustrate the relevance of the theoretical model by presenting information for the housing market in two cities. Section 6 summarize the main results and provides some ideas for further research.

2. Spatial segregation of single-person and multi-person households

The baseline model

We consider a mono-centric city inhabited by individuals with identical preferences. Some individuals live in single-person households, others in multi-person households. Households derive utility from a hicksian composite good \( c_n \) and from the lot of land \( q_n \) they are living on, with \( n \) indicating household size. Let \( v = v(c_n, q_n; n) \) be the household utility function. Since there may be economies of scale in household consumption, in particular for housing, household size enters the household utility function as a parameter. The utility function is assumed to be strictly quasi-concave in \( c_n \) and \( q_n \). The members of the household work a fixed number of hours at a fixed wage rate, thereby earning an (household income, \( y_n \), which conditional on household size can be treated as exogenous. The household spends its income on the composite good, on renting land, and on commuting to work in the spaceless centre of the city. Total costs of commuting amount to \( t_n x \), where \( x \) is commuting distance, and \( t_n \) is the cost of commuting one kilometre roundtrip for all members of a household of size \( n \). When the composite good is chosen as the numeraire, the budget constraint takes the form

\[
y_n = c_n + p_n(x)q_n + t_n x, \quad \text{where } p_n(x) \text{ is the rental cost per square metre of land located at distance } x \text{ from the city centre.}
\]

Substituting from the budget constraint into the utility function, the utility maximization problem of an \( n \)-person household living at distance \( x \) from the city centre takes the form:

\[
\max_{c_n, q_n} v(y_n - p_n(x)q_n - t_n x, q_n; n)
\]

(1)

This yields the first order condition:

\[
\frac{v_{c_n}}{v_{q_n}} (y_n - p_n(x)q_n - t_n x, q_n; n) = p_n,
\]

(2)

where \( v_{c_n} \) and \( v_{q_n} \) are the partial derivatives of the utility function.

We assume that individual preferences identical. It then follows that preferences for all households of a given size will be identical. Household of different size will, however, generally have different preferences. We also assume that all households of a given size have the same income and costs of transportation. Under these stylized assumptions, households of a given size living at different distances from the city centre must obtain the same utility. Households of different size will, however, in general differ in income and costs of transportation, and will enjoy different levels of utility. Hence, we have the restriction:
\[ v(y_n) - p_n(x)q_n - t_n, x, q_n; n) = \bar{v}_n \quad n \in [1, 2], \]

(3)

where the bar on the r.h.s. variable indicates that there is a common utility level for all households of a given size. Differentiating Eq. (3) w.r.t. \( x \) yields the rent gradient:

\[ \frac{\partial p_n}{\partial x} = -\frac{t_n}{q_n} \quad n \in [1, 2]. \]

(4)

Eq. (4) tells us that the rent gradient for land in equilibrium drops as one move away from the city centre. Consequently, households with high costs of commuting will in equilibrium be compensated through a lower land rent, so that all households of a given size will enjoy the same level of utility, irrespective of where in the city they live.

Modelling spatial segregation according to household size

As indicated by Eq. (4), the rent gradient will in general be different for households of different size. Since lots at a given location will be rented out to the highest bidder, the household type with the steepest rent gradient will occupy the central parts of the city. Consequently, if single-person households have a steeper rent gradient than two-person households, single-person households will in equilibrium bid up rents in the central parts of the city, and occupy the central part, while two-person households will be willing to pay the highest rents in the outer parts, and will occupy this part. By contrast, if single-person household have a less steep rent gradient than two-person households, single-person households will in equilibrium occupy the outer part of the city, and two-person households the inner part. In order to examine which of these two cases apply, consider the situation at the distance \( \bar{x} \) from the city centre where we find the borderline between the areas occupied by the two household types. At this location the rent-functions of the two types of households cross, and in equilibrium they will both have to pay the same rent per unit of land. Initially, let us also assume that the two households have the same income, and that their costs of transportation also are identical (these initial assumptions will be modified below). Preferences are, however, assumed to differ between household types as stated by the following assumption:

Assumption 1: Preferences satisfy the restriction \( MRS_n > MRS_1 \), where \( MRS_n = -\frac{v_n^2}{v_n} \) is the marginal rate of substitution between lot size and the numeraire good for a household of size \( n \in \{1, 2\} \).

Assumption 1 restricts preferences to be single-crossing, which are illustrated in Figure 1. Economies of scale in consumption provide in the present case a rationale for single-crossing preferences. At point A, \( v_2 \) is an indifference curve for a two-person household, while \( v_1 \) is an indifference curve for a single-person household. If the two households have the same income net of transportation costs, represented by the budget constraint FG, the optimum for the two-person household is at point A. By contrast, point B will maximize the utility of the single-person household. Hence, if the two households have the same incomes and costs of transportation, the single-person household will, with the type of single-crossing preferences stated in Assumption 1, consume less land than the two-person household. At distance \( \bar{x} \) from the city centre a single-person household will then have a steeper rent gradient than a two-person household. Consequently, single-person households will then occupy the inner part of the city.

The assumptions that households containing one and two individuals have the same income and costs of transportation are highly unrealistic. Hence, in the sequel the analysis will be based on two additional assumptions, the first of which is:

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2 For instance, Brueckner (1977) demonstrates that high-income households will settle on large floor space in the outskirts of the city, while low-income households will occupy the central parts of the city. In a similar vein, Solow (1971) demonstrates that business activity will take place in the central parts of the city, while the residential areas will occupy the less central parts.
Assumption 2: Each adult person earns the same income, y.

Assumption 2 implies that income per capita in the city will be independent of city population and of how individuals group themselves together in households. At least as a first approximation, this seems reasonable. Moreover, since Wheaton (1974), Brueckner (1987) and many others have found that income has a profound impact on the equilibrium of the urban housing sector, we want in the present paper to neutralize this source of difference between cities. Assumption 2 serves this purpose.

Figure 1 Single-crossing preferences for households of different size.

In order to be able to make precise inference of how the income difference between households of different size affects the consumption of housing, we also incur the following assumption, which is frequently made in different variants of the mono-centric urban model:

Assumption 3: Housing is a normal good

Next, let us consider costs of transportation. It is reasonable to assume that these differ systematically between households of different size. If each individual in the household holds a full job, and if all costs of transportation to the city-centre were related to job-commuting, it would be reasonable to assume \( t_2 = 2t_1 \). Households incur, however, costs of transportation also when they travel to the city centre for shopping, or for visiting cultural amenities, restaurants, etc. When measured on a per
person basis, such transportation costs are likely to be much higher for individuals living as singles than for individuals in two-person households. One reason for this is that one of the two individuals in a two-person household often may do shopping for the whole household. Another reason is that a single-person household is likely to incur higher commuting costs than a two-person household in order to obtain social contact with other individuals at restaurants, cultural amenities, etc. For a two-person household much of the social contact is obtained within the household, without any costs of transportation. Based on these arguments, we assume:

Assumption 4: The costs of transportation for households of size 1 and 2 are related as follows

\[ t_1 = \alpha t_2, \text{ where the parameter } \alpha \in (0.5,1). \]

Assumptions 2 and 4 combined imply that income net of transportation costs for a single-person household living at distance \( \bar{x} \) from the city centre will be less than half of what it is for a two-person household at the same location. The budget constraint of a single-person household is in Figure 1 shown as the line IJ.

Under Assumptions 1, 2, 3, and 4 the optimum of a single-person household living at distance \( \bar{x} \) from the city centre will be at a point like C in Figure 1, while it for a two-person household will be at A. From Figure 1 we can then conclude that the consumption of land for a single-person household compared to a two-person household, both living at distance \( \bar{x} \) from the city centre, is affected negatively by the difference in preferences as well as by the drop in income net of transportation costs.

The analysis so far tells us that a single-person household living at distance \( \bar{x} \) from the city centre under reasonable assumptions will have a lower consumption of land than a two-person household. However, since the single-person household also has lower transportation costs, additional restrictions are needed to determine which household type will have the steepest rent gradient at \( \bar{x} \). For this purpose, let us rewrite the rent-gradient for a single-person household as:

\[
\frac{\partial p_1}{\partial x} = \left( \frac{-1}{q_2} \right) \left( \frac{\alpha}{(1-\beta_1)(1-\beta_2)} \right),
\]

where \( \beta_1 \in [0,1] \) is the relative reduction in demand for land due to different preferences if household size drops from 2 to 1, i.e. the move from A to B in Figure 1. Next, \( \beta_2 \in [0,1] \) is the relative drop in demand for land due to Assumption 2, i.e. that a single-person household has a lower income than a two-person household. If we in Eq. (5) have \( \gamma > 1 \), a single-person household will have a steeper rent gradient than a two-person household. This yields:

**Proposition 1.** If \( \alpha > (1-\beta_1)(1-\beta_2) \), single-person households have at \( \bar{x} \) a steeper rent gradient than two-person households. When this condition is fulfilled, single person households will occupy the dwellings closer to the city centre than \( \bar{x} \), while two-person households will live further from the city centre than \( \bar{x} \).

In order to assess the role of the condition in Proposition 1, let us plug in the borderline values \( \alpha = 0.5 \) and \( \beta_1 = 0 \). We then obtain the condition \( \beta_2 > 0.5 \). Combined with our assumption of how household income changes when a two-person household is split in two, \( \beta_2 > 0.5 \) implies that the income elasticity, \( E^q \), of the demand for land must be at least 1.0. This condition is likely to be fulfilled. In fact, Chishire (98) obtained estimates of the income elasticity of demand for lot size in the interval 1.47-1.82. With an income elasticity within this range, the condition in Proposition 1 is fulfilled.

\(^3\)At the cost of a substantially more complex model it would be possible to include social contact as a basic good in the utility function and to let the consumer choose endogenously the volume of this good, as well as the necessary costs of transportation in producing social contact. The simpler approach taken in the present paper corresponds well, however, to the assumption that work participation as well as working hours and number of commuting trips are taken as exogenous.
even with the extreme assumptions $\alpha = 0.5$ and $\beta_1 = 0$. Choosing more realistic assumptions for $\alpha$ and $\beta_1$, say $\alpha = 0.6$ and $\beta_1 = 0.1$, the condition in proposition 2 will be satisfied for $\beta_2 > 0.33$, which is satisfied for $E^d \geq 0.27$. Hence, we conclude that the condition stated in Proposition 1 is unlikely not to be satisfied. Single-person households are therefore in equilibrium likely to outbid two-person households for the dwellings closer to the city centre than $\bar{x}$. This situation is illustrated in Figure 2. The analysis in the sequel is based on this.

![Figure 2 Spatial segregation of single-person and two-person households](image)

3. Empirical evidence

The theory developed in the previous section predicts that 100 per cent of the households in the central parts of a city should be single-person households, while the percentage should be 0 in the outer parts of the city. These predictions are, however, derived from a very stylized model. In real-world cities, many other factors than those included in our model may have also have an impact of the share of singles in different parts of the city. Nevertheless, as a first approach to an empirical investigation of our model, it might be useful to consider how the share of single-person households varies with location within real-world cities. For this purpose, I have selected 2 cities: Oslo and Munich.

Oslo is the capital city of Norway. The municipality of Oslo had, according to the Population Census (2001), slightly more than 510,000 inhabitants. The Oslo urban area extends beyond the borders of the municipality, but in the following we confine interests to the municipality. Using data from the Population Census (2001) we have constructed the map in Figure 3. It shows how the share of single-person households varies considerably between the 27 districts of Oslo municipality. In the Central Business District (CBD), in the core of the dark area in Figure 3, 75 per cent of the households consist of a single person. Around the CBD there is a belt where 60 -70 per cent of the households consists of...
single persons. At greater distance from the CBD, there is a belt where 40-50 per cent of the households contain only one single person. Although there are a few exceptions from the general pattern of concentric rings, the pattern is striking. Notice also that we can discern a kind of dichotomy: in the CBD and the belt immediately around it, the share of single-person households is 60 per cent or more. In the outer parts of the city, by contrast, the share of single-person households is in general between 40-50 percent. Only in one district is the share between 50 and 60 percent.

Munich is one of the largest towns in Germany, and had in 2008 about 1 367 000 inhabitants. From Figure 4 it can be seen that this city comes close to the circular tow assumed in our theoretical model. Figure 4 displays the 25 parts (Stadtbezirken) in which Munich is divided, along with the share of single-person households for each part of the city. We see that more than 60 per cent of the households living in the inner parts of the city are single-person households. By contrast, less than 50 percent of the households in the outer parts of the city consist of a single person. Hence, there is a clear pattern with single-person households concentrated in the central parts of the city.

Figure 3 Oslo

Legend:
- Black area: Share of single-person households exceeds 60 per cent
- Dotted area: Share of single-person households less than 60 per cent. Densely dotted when the share is close to 60 per cent. In the most sparsely dotted areas the share is close to 30 per cent.

Figur 4 Munich

Legend:
- Black area: Share of single-person households exceeds 60 per cent
- Dotted area: Share of single-person households less than 60 per cent. Densely dotted when the share is close to 60 per cent. In the most sparsely dotted areas the share is close to 30 per cent.

Although neither Oslo nor Munich reveal a strict dichotomy of areas of residence for single-person and multi-person households as predicted by the theory, the empirical evidence clearly suggests that household size is an important determinant for where in these cities households of different size reside. The two case reviewed clearly indicate that single-person households are concentrated in the most central parts of cities. The strict dichotomy implied by our theoretical model is not observed. It is,
however, important to recall that our theoretical model is highly stylized. Moreover, the theoretical model should be interpreted as a picture of how a city would be in a long-run equilibrium. At a specific point in time, the existing buildings in a real-world city have been built over a long period of time. Hence, they reflect equilibrium conditions at earlier points in time. Second, when individuals initially living as singles marry or become cohabitants, they often live in the dwelling that previously accommodated one of them. Over time, however, they are likely to move to a larger dwelling located less centrally, in particular if they get children. Third, when multi-person households split up due to divorce, one of the adults and the children often continue to live in the original dwelling, at least for some time

4. Concluding remarks

Within a framework where all individuals have identical preferences, but where economies of scale gives rise to different single-crossing preference maps for single-person and two-person households, we demonstrated that single-person households under reasonable assumptions will occupy the central parts of cities, while two-person households will live in the outer parts. It is noteworthy that these results basically follow from economies of scale in households and competition for land.

References


